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# Charmonium Suppression and Regeneration from SPS to RHIC

L. Grandchamp<sup>1,2,\*</sup> and R. Rapp<sup>1</sup>

<sup>1</sup>*Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, NY 11794-3800*

<sup>2</sup>*Institut de Physique Nucléaire de Lyon, 43 Bd du 11 Novembre, 69622 Villeurbanne cedex*

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The production of charmonia is investigated for heavy-ion collisions from SPS to RHIC energies. Our approach incorporates two sources of  $J/\Psi$  yield: (i) a direct contribution arising from early (hard) parton-parton collisions, subject to subsequent nuclear absorption, quark-gluon plasma and hadronic dissociation, and (ii) statistical production at the hadronization transition by coalescence of  $c$  and  $\bar{c}$  quarks. Within an expanding thermal fireball framework, the model reproduces  $J/\Psi$  centrality dependencies observed at the SPS in  $Pb-Pb$  and  $S-U$  collisions reasonably well. The study of the  $\Psi'/\Psi$  ratio at SPS points at the importance of the hadronic phase for  $\Psi'$  interactions, possibly related to effects of chiral symmetry restoration. Predictions are given for the centrality dependence of the  $N_{J/\Psi}/N_{c\bar{c}}$  ratio at full RHIC energy. We also calculate the excitation function of this ratio. The latter exhibits a characteristic minimum structure signalling the transition from the standard  $J/\Psi$  suppression scenario prevailing at SPS to dominantly thermal regeneration at collider energies.

## I. INTRODUCTION

The main objective of ultrarelativistic heavy-ion physics, both experimentally and theoretically, is to create and study hot and dense matter of strongly interacting constituents. At sufficiently large temperatures and densities, the theory of strong interactions (QCD) predicts a phase transition from the hadronic world into a chirally symmetric state of free quarks and gluons, the so-called Quark-Gluon Plasma (QGP). However, the transient nature of this new state of matter renders its identification very complex. After more than a decade of experiments, a wealth of exciting results has emerged from the SPS program as well as from recent data taken at the the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven. However, as of now, no conclusive evidence of QGP formation has been discerned. It has rather become clear that the discovery of the QGP would not emerge from a single signature but from a combination of several in a comprehensive scenario.

Among the probes of QGP formation, the  $J/\Psi$  meson plays a central role. Due to its large binding energy and rather small size, it is expected to be only mildly affected by interactions in a hadronic environment. On the contrary, as suggested first by Matsui and Satz [1] in 1986, if a QGP is formed,  $J/\Psi$  formation should be inhibited due to color charge screening by free-moving quarks and gluons. From a slightly different perspective, a similar effect can be attributed to parton-induced destruction [2]. In either picture, the  $J/\Psi$  abundance observed in heavy-ion collisions is linked to properties of the initial hot and/or dense phases of the produced matter. However, the identification of the plasma effect requires a careful assessment of other mechanisms for  $J/\Psi$  destruction in the course of a heavy-ion reaction. Most notably, absorption on high-energetic nucleons at early times induces a substantial suppression of typically 60% in central collisions. Although its dependence has been thoroughly investigated experimentally in both proton and light-ion induced reactions with various targets [3, 4, 5, 6] theoretical uncertainties in the application to the heavy-projectile case persist [7]. Furthermore, despite the naive expectation given above, the current status in the description of inelastic charmonium scattering on “comoving” (secondary) hadrons in the late stages of the collision is not satisfactorily under control. This renders the identification of the effect of the quark-gluon plasma formation on  $J/\Psi$  abundances more difficult to establish.

Recently, an additional source of  $J/\Psi$  production in heavy-ion collisions has been suggested. Within thermal model frameworks [8, 9, 10], which successfully describe light-hadron ratios, statistical production of charmonium at the QCD hadronization transition has been evaluated [11, 12, 13, 14]. Gazdzicki and Gorenstein found [11] that statistical production alone can account for the observed centrality dependence of  $J/\Psi$  yields at full SPS energy, thereby deducing a hadronization temperature consistent with standard chemical freezeout. However, no reference was made to a dynamical origin for  $c\bar{c}$  creation, nor to open-charm states. Braun-Munzinger and Stachel [12, 13] started from the dynamically well justified [15] hypothesis that at SPS energies all  $c\bar{c}$  pairs are exclusively created in primordial (hard) parton-parton collisions. Open- and hidden-charm hadrons are then populated at the hadronization

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\*Electronic address: loic@tonic.physics.sunysb.edu

transition according to their thermal weight including a charm-quark fugacity to match the primordial  $c\bar{c}$  abundance. With the latter taken from extrapolations of  $N$ - $N$  collisions, a substantial fraction of  $J/\Psi$  mesons is found. In particular, the  $\Psi'$  over  $\Psi$  ratio, which in  $Pb$ - $Pb$  reactions has been observed to rapidly approach its thermal value for  $N_{part} \geq 100$ , is well described. Along similar lines, Gorenstein *et al.* [14] aim at explaining the  $J/\Psi$  yields at SPS solely in terms of statistical production. They conclude that an open-charm enhancement factor of up to  $\sim 6$  in central  $Pb$ - $Pb$  collisions (w.r.t. the standard value inferred from  $N$ - $N$  collisions) is required to optimally reproduce the NA50 data [16, 17]. Finally, Thews *et al.* [18] assessed  $c\bar{c}$  coalescence in an expanding QGP fireball by solving rate equations for  $J/\Psi + g \leftrightarrow c\bar{c}$  reactions (gluon-induced “photo”-dissociation and its reverse reaction), with the  $J/\Psi$  binding energy assumed to be at its vacuum value at all times in the evolution.

All thermal approaches share the common feature that, as first pointed out in Ref. [12], at RHIC energies a copious production of charm-quark pairs ( $N_{c\bar{c}} \simeq 10$ -20 in central  $Au$ - $Au$  collisions) implies a much enhanced charmonium yield as compared to early (hard) production coupled with nuclear and QGP suppression mechanisms. However, at SPS energies, expected plasma lifetimes of  $\sim 1$ -2 fm/c, together with initial temperatures below  $\sim 250$  MeV, may not suffice for a (close to) complete destruction of primordially produced  $J/\Psi$  mesons. At the same time, a significant increase in open-charm production is not easily justified. This led us to propose a two-component model [19] for charmonium production in heavy-ion collisions. It incorporates both a primordial yield (subject to subsequent dissociation) as well as a thermal contribution from statistical recombination of  $c$  and  $\bar{c}$  quarks at the hadronization transition. Both components are evaluated within a common thermal fireball framework which is consistent with measured hadro-chemistry, expansion dynamics, and has also been successfully employed to describe electromagnetic observables at SPS energies [20]. Another important feature is that we refrain from invoking any “anomalous” open-charm enhancement.

In the present article, we expand upon our previous analysis in several respects: First, and most importantly, we give a detailed account of centrality dependencies for  $J/\Psi$  yields in both  $S$ - $U$  and  $Pb$ - $Pb$  systems at SPS, as well as predictions for  $Au$ - $Au$  at full RHIC energy. The relevance of hadronic dissociation is assessed, especially in the context of the  $\Psi'/\Psi$  ratio measured at SPS. Higher charmonium states relevant for feeddown contributions to the  $J/\Psi$  are included on an equal footing. Effects of incomplete charm-quark thermalization, which affect thermal recombination, are incorporated on a phenomenological basis. We also lay out further steps to investigate problems that will not be satisfactorily addressed here.

The paper is organized as follows: In Sect. II, we recall basic elements of nuclear absorption including normalization issues when comparing to NA38 and NA50 data. In Sect. III, we present our calculations for the charmonium dissociation processes, *i.e.*, parton-induced destruction in a QGP as well as inelastic hadronic  $J/\Psi$  interactions using effective Lagrangians. Pertinent survival probabilities are obtained from a convolution of the destruction rates over the space-time history of a collision modeled within an expanding fireball. Sect. IV is devoted to the description of statistical charmonium formation at the hadronization transition including exact charm conservation constraints. In Sect. V, we combine the two sources of charmonia to investigate the  $J/\Psi$  centrality dependence and  $\Psi'/\Psi$  ratio at SPS for both  $S(200 \text{ AGeV})$ - $U$  and  $Pb(158 \text{ AGeV})$ - $Pb$  systems. In Sect. VI, we give predictions for the ratio  $N_{J/\Psi}/N_{c\bar{c}}$  as a function of centrality at RHIC, and discuss the excitation function of this ratio from SPS to RHIC energies. We summarize and conclude in Sect. VIII, and indicate directions for future improvements.

## II. NUCLEAR ABSORPTION OF CHARMONIA

A suppressed abundance of  $J/\Psi$  mesons relative to expectations from  $N$ - $N$  collisions was first observed in proton-nucleus ( $p$ - $A$ ) and light-ion induced collisions. In these reactions, one does not expect noticeable effects from secondary particles. Indeed, rather extensive experimental studies [3, 4, 5, 6] have established that the suppression can be understood as the absorption of a “pre-resonance”  $c\bar{c}$  state in the (normal) nuclear medium of target and projectile nuclei. This in particular accounts for the observation that different charmonium states such as  $J/\Psi$  and  $\Psi'$  follow very similar absorption patterns, despite their different bound state properties (*e.g.*, binding energy or size).

Let us briefly summarize the main elements of this “nuclear absorption”. In the Glauber model of  $A$ - $B$  collisions, the probability for the pre-resonance state to be absorbed on its way through the nucleus is given by [21]

$$\begin{aligned} \mathcal{S}_{nuc}(b, \sigma_{nuc}) &= \frac{1}{T_{AB}(b)} \int d^2s \, dz \, dz' \, \rho_A(\vec{s}, z) \, \rho_B(\vec{b} - \vec{s}, z') \\ &\times \exp \left\{ -(A-1) \int_z^\infty dz_A \rho_A(\vec{s}, z_A) \sigma_{nuc} \right\} \times \exp \left\{ -(B-1) \int_{z'}^\infty dz_B \rho_B(\vec{b} - \vec{s}, z_B) \sigma_{nuc} \right\} \end{aligned} \quad (1)$$

where  $\rho_A(\vec{s}, z)$  describes nuclear density profiles (taken from Ref. [22]),  $\vec{s}$  the position of the  $c\bar{c}$  production point in the transverse plane, and  $b$  the impact parameter. The coordinates  $z$  and  $z'$  specify the positions within nucleus  $A$

and  $B$ , respectively, along the collision axis.  $T_{AB}(b)$  is the usual nuclear overlap function of the two colliding nuclei. The pre-resonance “absorption” cross section  $\sigma_{nuc}$  is treated as a free parameter being adjusted to experimental data.

In the NA38/NA50 experiments, from which all data shown below are taken, the main measure for the centrality of a nuclear collisions is given by the transverse energy  $E_T$  detected in the calorimeter. The impact parameter  $b$  is commonly related to this observable through the wounded nucleon model, which, upon inclusion of fluctuations in  $E_T$  at fixed  $b$ , provides a correlation function

$$\mathcal{P}_{AB}(E_T, b) = \frac{1}{\sqrt{2\pi q^2 a N_{part}(b)}} \exp \left\{ -\frac{[E_T - q N_{part}(b)]^2}{2q^2 a N_{part}(b)} \right\}. \quad (2)$$

Here,  $q$  is a proportionality factor between  $E_T$  and the number of participant,  $E_T(b) = q N_{part}$ , which depends on the specific experimental settings, and  $a$  is an empirical parameter characterizing the magnitude of the fluctuations. The absorption of charmonium in nuclear matter as a function of  $E_T$  then follows as

$$\frac{B_{\mu\mu} \sigma_{J/\Psi}}{\sigma_{DY}} = \frac{B_{\mu\mu} \sigma_{J/\Psi}^{pp} \int d^2b \mathcal{P}_{AB}(E_T, b) \mathcal{S}_{nuc}(b, \sigma_{nuc}) T_{AB}(b)}{\sigma_{DY}^{pp} \int d^2b \mathcal{P}_{AB}(E_T, b) T_{AB}(b)} \quad (3)$$

where  $B_{\mu\mu}$  is the branching ratio for  $J/\Psi \rightarrow \mu^+ \mu^-$ . The pre-factor  $\sigma_{J/\Psi}^{pp}/\sigma_{DY}^{pp}$  is not accurately known, and depends on the energy of the collision.

At SPS energies, we will concentrate on the two collision systems  $S(200 \text{ AGeV})$ - $U$  and  $Pb(158 \text{ AGeV})$ - $Pb$ . For  $q$  and  $a$  we use the values reported by the experiments, *i.e.*,  $q = 0.275$  (0.72) and  $a = 1.27$  (1.56) for  $Pb$ - $Pb$  ( $S$ - $U$ ). Together with an absorption cross section of  $\sigma_{nuc} = 6.4 \text{ mb}$  and normalization factors of 52.8 for  $Pb$ - $Pb$  and 48.2 for  $S$ - $U$ , our results coincide with nuclear absorption studies of NA38 and NA50, cf. Fig. 1. Whereas the  $S$ - $U$  data are well accounted for, the  $Pb$ - $Pb$  system departs from nuclear absorption starting at  $E_T \simeq 40 \text{ GeV}$ , constituting “anomalous”  $J/\Psi$  suppression [23, 24, 25, 26].

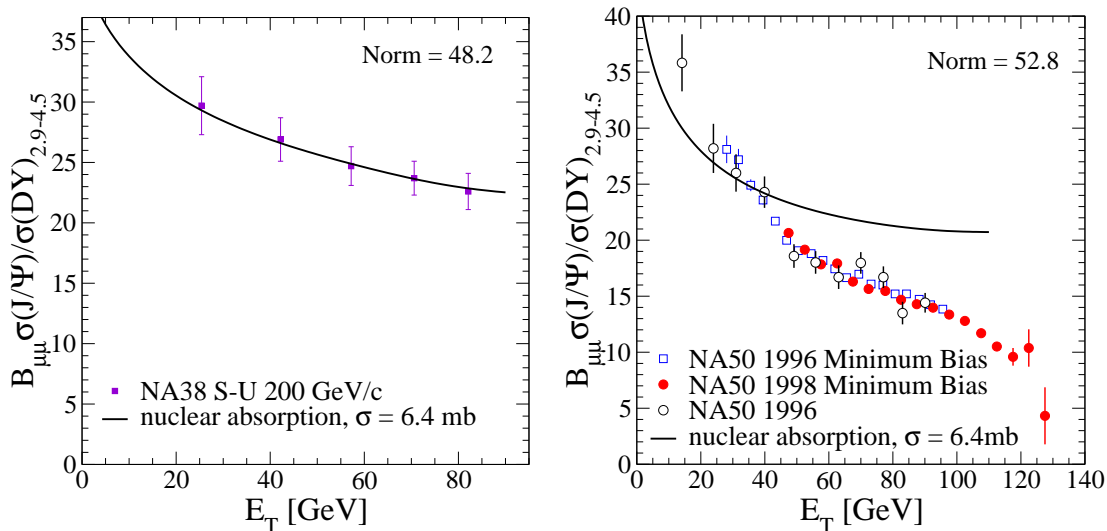


FIG. 1: Nuclear absorption calculations for the  $S(200 \text{ AGeV})$ - $U$  (left panel) and  $Pb(158 \text{ AGeV})$ - $Pb$  (right panel) systems at the CERN-SPS, compared to data from NA38 [5] and NA50 [26], respectively. The normalization factor  $B_{\mu\mu} \sigma_{J/\Psi}^{pp}/\sigma_{DY}^{pp}$  has been fixed at 48.2 (52.8) for the  $S$ - $U$  ( $Pb$ - $Pb$ ) system.

### III. $J/\Psi$ DISSOCIATION IN HEAVY-ION COLLISIONS

The deviation of the  $J/\Psi$  yield from nuclear absorption systematics in  $Pb(158 \text{ AGeV})$ - $Pb$  has triggered extensive theoretical analyses. The responsible underlying mechanisms are still a matter of debate, ranging from destruction in a QGP and/or hadron gas to thermal production at  $T_c$ . As stressed above, this work attempts a comprehensive treatment of all three of these aspects within a minimal set of assumptions, which, in particular, is based on a thermal

description of the collision dynamics including Quark-Gluon Plasma formation if the initial conditions are energetic enough. In this section, we compute  $J/\Psi$  survival probabilities for both the plasma and hadronic phases of heavy-ion reactions.

### A. Quark-Gluon Plasma

$J/\Psi$  dissociation in a thermalized QGP has been studied in both static and dynamical frameworks. Within the former, one typically evaluates the screening of the heavy quark potential by color charges, whereas the latter involves inelastic parton collisions using, *e.g.*, the QCD analogue of photo-dissociation,  $g + J/\Psi \rightarrow c + \bar{c}$  [27].

Here we adopt a dynamical approach, accounting however for reduced charmonium binding energies as extracted from a Schrödinger equation for  $c\bar{c}$  bound states in a screened heavy-quark potential [28]. Under conditions relevant to the initial stage of heavy-ion collisions, the binding energy of the  $J/\Psi$  is strongly reduced with respect to its vacuum value, which is also borne out of recent lattice gauge calculations [29]. Color-screening is characterized by the electric Debye mass  $\mu_D$  which we estimate to leading order in perturbation theory,  $\mu_D \sim gT$ . Since, strictly speaking, perturbation theory is not really applicable under the moderate plasma temperatures expected even at RHIC energies, we regard the strong coupling constant as an effective parameter to be adjusted to the  $J/\Psi$  data at SPS. *E.g.*, with a typical  $g \simeq 1.7$ , the  $J/\Psi$  binding energy at  $T = 170$  MeV is  $E_{diss} = 250$  MeV decreasing to  $E_{diss} = 100$  MeV at  $T = 230$  MeV and vanishing around  $T \simeq 360$  MeV. For such small binding energies, the photo-dissociation process becomes inefficient due to unfavorable break-up kinematics. For a loosely bound charmonium state, inelastic parton scattering,  $g(q, \bar{q}) + J/\Psi \rightarrow g(q, \bar{q}) + c + \bar{c}$ , turns out to be a more important mechanism [19]. The respective cross sections are evaluated in quasifree approximation using leading-order QCD for  $gc \rightarrow gc$  ( $qc \rightarrow qc$ ) [30]. The thermal dissociation rate is then obtained via

$$\Gamma_{diss} = \sum_{i=q,g} \int_{k_{min}}^{\infty} \frac{d^3k}{(2\pi)^3} f^i(k, T) \sigma_{diss}(s) \quad (4)$$

where  $k_{min}$  denotes the minimal on-shell momentum of a quark or gluon from the heat bath necessary to dissolve an in-medium charmonium bound state into a free  $c\bar{c}$  pair. We include thermal quasiparticle masses for light quarks and gluons [31],

$$m_{u,d}^2 = \frac{g^2 T^2}{6}, \quad m_s^2 = m_0^2 + \frac{g^2 T^2}{6}, \quad m_g^2 = \frac{g^2 T^2}{2}. \quad (5)$$

This formalism straightforwardly applies to  $\Psi'$  and  $\chi$  states as well. From the Schrödinger equation, it follows [28] that their binding energy vanishes at or even below  $T_c$ , which is also found in recent lattice calculations [29]. Therefore, the pertinent QGP dissociation rate is given by Eq. (4) with  $E_{diss} = 0$ . The picture we have in mind here is similar in spirit to that underlying nuclear absorption: a pre-resonance state, characterized by a correlation induced in the primordial  $N$ - $N$  collision (*e.g.*, comoving  $c$  and  $\bar{c}$  quarks), is only destroyed if it actually undergoes an inelastic interaction with the surrounding matter.

The resulting charmonium dissociation times,  $\tau_{diss} = \Gamma_{diss}^{-1}$ , are shown in Fig. 2 and, for the  $J/\Psi$ , are compared to photo-dissociation *without* medium effects in the bound state energy (as has been employed in the literature before). For temperatures relevant at SPS, the quasifree process (full line) is more efficient in dissolving  $J/\Psi$ 's than photo-dissociation (dotted line). At higher temperatures this tendency is reversed, mostly due to an increasing Debye mass which suppresses the  $t$ -channel gluon-exchange graphs for  $g(q, \bar{q}) + J/\Psi \rightarrow g(q, \bar{q}) + c + \bar{c}$ . Close to  $T_c$ , the vanishing binding energies for  $\Psi'$  and  $\chi$  entail lifetimes which are about a factor of three below the one for  $J/\Psi$ -mesons.

### B. Hadron Gas

Subsequent to quark-gluon plasma dissociation, when the system converts to the hadronic phase, charmonia undergo further suppression due to interactions with surrounding hadrons. There are a number of theoretical models for  $J/\Psi + \text{hadron}$  processes [32, 33, 34, 35, 36, 37, 38, 39] whose results span an appreciable magnitude. Calculations involving excited charmonia, such as  $\Psi'$  and  $\chi_c$  (which contribute to the  $J/\Psi$  yield via electromagnetic feeddown), are scarce. Consequently, the impact of inelastic hadronic scattering is not very well under control. Most calculations, including recent ones based on quark exchange models [38] or on  $SU(4)$ -symmetric effective lagrangians [37, 39], seem to indicate a rather moderate impact of the hadronic medium on the  $J/\Psi$ .

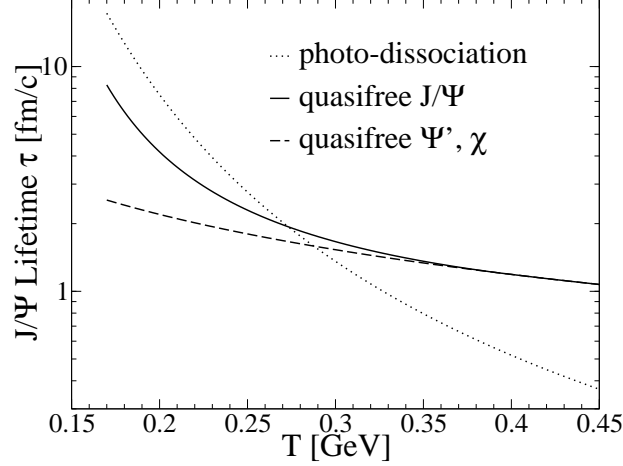


FIG. 2: Dissociation times for a  $J/\Psi$  ( $\Psi'$ ,  $\chi$ ) in a QGP as a function of temperature. The full (dashed) curve corresponds to the leading-order QCD process for quasifree  $g, q + c \rightarrow g, q + c$  scattering with *in-medium*  $J/\Psi$  ( $\Psi'$ ,  $\chi$ ) bound state energies. The dotted curve results from photo-dissociation,  $gJ/\Psi \rightarrow c\bar{c}$ , assuming the *vacuum* dissociation energy.

### 1. Absorption on light hadrons

As a baseline calculation, we here reproduce results obtained in Refs. [37, 39] within a  $SU(4)$  effective theory. The starting point is a  $SU(4)$ -flavor symmetric effective lagrangian formulated with 4-by-4 pseudoscalar and vector meson matrices. Although the  $SU(4)$  symmetry is strongly broken by the  $c$ -quark mass, the hope is that symmetry-breaking effects are largely accounted for by the hadronic mass matrix. Along these lines we compute (inelastic) interactions of the  $J/\Psi$  with pions ( $\pi + J/\Psi \rightarrow D + \bar{D}^*$ ,  $\bar{D} + D^*$ ) and rhos ( $\rho + J/\Psi \rightarrow D + \bar{D}$ ,  $\rho + J/\Psi \rightarrow D^* + \bar{D}^*$ ) which are the most abundant mesons in the medium. We employ coupling constants as calibrated by Haglin and Gale in Ref. [39] based on the  $\rho \rightarrow \pi\pi$  decay to fix the gauge coupling. The effective hadronic theory is supplemented by vertex form factors to simulate finite-size effects. This violates gauge invariance, which, however, can be restored by introducing appropriate counter terms [39]. We have checked for various processes (*e.g.*,  $\pi + J/\Psi \rightarrow D + \bar{D}^*$ ) that these extra terms induce only quantitatively minor modifications, which we therefore neglected. The corresponding  $J/\Psi$  lifetime in a  $\pi$ - $\rho$  gas is displayed in Fig. 3 using (covariant) monopole form factors with cutoff  $\Lambda = 1$  GeV.

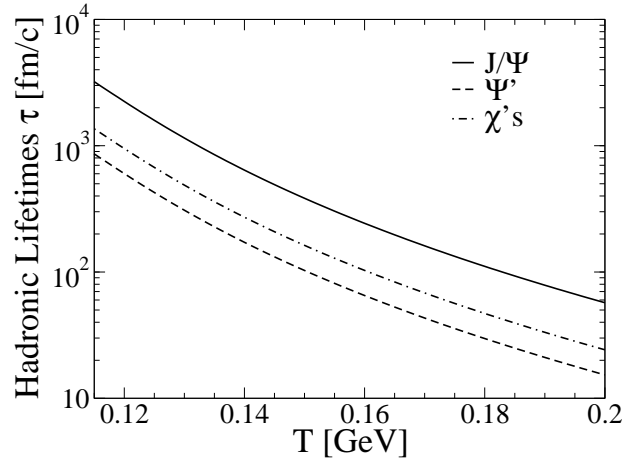


FIG. 3: Hadronic lifetimes for  $J/\Psi$  (solid line),  $\Psi'$  (dashed line) and  $\chi$  (dot-dashed line) based on a  $SU(4)$  effective lagrangian, including interactions with pions and rhos using a form factor value of  $\Lambda = 1$  GeV. The curves for the  $\Psi'$  ( $\chi$ ) are deduced from the  $J/\Psi$  one via cross section scaling by  $(r_{\Psi'}/r_{J/\Psi})^2$  ( $(r_{\chi}/r_{J/\Psi})^2$  for the  $\chi$ ).

As mentioned above,  $\Psi'$  and  $\chi$  dissociation rates are even more difficult to assess within hadronic model frameworks. To obtain an estimate that can be used in our calculations below, we assume a geometric scaling of the calculated  $J/\Psi$  cross sections with the squared ratio of the respective charmonium radii [28],  $(r_{\Psi'}/r_{J/\Psi})^2$  and  $(r_{\chi}/r_{J/\Psi})^2$ , cf. dashed

and dot-dashed lines in Fig. 3. It turns out that these estimates agree reasonably well with explicit calculations of the corresponding processes within the constituent quark model approach of Ref. [38].

## 2. Anomalous Processes

We furthermore investigated the role of so-called anomalous processes such as  $\pi + J/\Psi \rightarrow \eta_c + \rho$ ,  $\pi + J/\Psi \rightarrow \eta_c + b_1$ , and  $\rho + J/\Psi \rightarrow \eta_c + \pi$ , suggested in Ref. [39]. In there, the relevant hadronic coupling constants, *e.g.*,  $g_{J/\Psi\omega\eta_c}$ , have been estimated applying the vector dominance model (VDM) to the radiative decay  $J/\Psi \rightarrow \gamma\eta_c$ . We believe, however, that this procedure leads to a significant overprediction of the hadronic coupling, for the following reason. For vertices carrying identical quantum number structure, namely  $J/\Psi\omega\eta$  and  $J/\Psi\omega\eta'$ , both hadronic and corresponding radiative decay information is available from experiment. One finds [40],

$$\begin{aligned}\Gamma(J/\Psi \rightarrow \omega\eta)/\Gamma_{tot} &= 1.6 \times 10^{-3}, & \Gamma(J/\Psi \rightarrow \omega\eta')/\Gamma_{tot} &= 1.7 \times 10^{-4}, \\ \Gamma(J/\Psi \rightarrow \gamma\eta)/\Gamma_{tot} &= 8.6 \times 10^{-4}, & \Gamma(J/\Psi \rightarrow \gamma\eta')/\Gamma_{tot} &= 4.3 \times 10^{-3},\end{aligned}$$

*i.e.*, the hadronic branching ratios are comparable or even below the radiative ones. This is in marked contradiction to VDM within which the latter are suppressed by a factor  $(e/g_\omega)^2$  which is much smaller than the moderate increase in phase space due to the final state decay momenta (also note that VDM is more strongly violated with increasing mass of the pseudoscalar meson). Thus it appears that VDM cannot be applied, and that an accordingly reduced  $g_{J/\Psi\omega\eta_c}$  coupling renders the pertinent  $t$ -channel  $\omega$  exchange processes in  $\pi + J/\Psi \rightarrow \eta_c + \rho$  and  $\pi + J/\Psi \rightarrow \eta_c + b_1$  negligible. We therefore decided not to include anomalous processes in our analysis.

## C. $J/\Psi$ survival probability in a thermal expansion scenario

The final number of primordial  $J/\Psi$ 's remaining after plasma and hadronic phases requires the convolution of the dissociation rates derived in the previous two sections over the space-time history of a given heavy-ion collision. To this end, we model this evolution by a schematic thermal fireball expansion [41], which, however, incorporates essential features of hydrodynamical calculations. Let us briefly recall the main elements.

Equilibration of the system is assumed at a formation time  $\tau_0$ , after which isentropic expansion proceeds at fixed entropy per baryon,  $S/N_B = s/n_B$ , where the total entropy  $S = sV_{FB}$  and net baryon number  $N_B = n_B V_{FB}$  are related to the pertinent densities via the time-dependent 3-volume  $V_{FB}$ . The latter is modeled in cylindrical symmetry as

$$V_{FB}(\tau) = 2(z_0 + v_z\tau + \frac{1}{2}a_z\tau^2)\pi(r_0 + \frac{1}{2}a_\perp\tau^2)^2 \quad (6)$$

(the overall prefactor of 2 accounts for two fireballs which enable to cover about 4 units of rapidity).  $r_0$  denotes the initial transverse overlap of the two colliding nuclei at given impact parameter  $b$ , whereas the expansion parameters  $\{v_z, a_z, a_\perp\}$  are adjusted in line with hydro-calculations to reproduce observed flow velocities in connection with (thermal) freezeout times of  $\tau_{fo} \simeq 10 - 14$  fm/c. The parameter  $z_0$  is equivalent to the formation time  $\tau_0$  (in the Bjorken limit  $z_0 \simeq \tau_0\Delta y$ ), specifying the initial conditions of the evolution. The temperature of the expanding matter is inferred from the entropy density  $s(\tau) = S/V_{FB}(\tau)$  in either hadronic or QGP phase (calculated from an ideal resonance gas for the former and using massive quasiparticles according to Eqs. (5) for the latter). If  $s(\tau)$  lies in between the entropy densities  $s_c^H$  and  $s_c^{QG}$  for hadron gas and QGP, a standard mixed phase construction [42] is employed at the critical temperature  $T_c$ ,

$$\frac{S}{V_{FB}(t)} = f s_c^H + (1 - f) s_c^{QG}, \quad (7)$$

which determines the volume partitions  $f$  and  $1 - f$  for hadronic and quark-gluon matter, respectively.

The collision-energy dependence of the underlying parameters from SPS to RHIC has been fixed as follows. The total entropy at given collision energy and centrality is obtained from the specific entropy  $S/N_B$  ranging from 26 to 250 from SPS to RHIC according to standard chemical freezeout analyses [8, 9] (with effectively conserved numbers of pions, kaons etc., thereafter). The critical temperature is assumed to increase smoothly from 170 MeV at SPS to 180 MeV at RHIC (with  $\mu_B$  decreasing from 260 to 27 MeV). In addition, we account for 10-20 % variations in  $S/N_B$  with centrality following particle production systematics reported by NA49 [43] and PHOBOS [44]. Finally, the formation time  $\tau_0$  (*i.e.*,  $z_0$ ) is continuously decreased from 1 to 1/3 fm/c for  $\sqrt{s_{NN}}$  from 17 to 200 GeV. With

the volume expansion as given by Eq. (6), this results in QGP phase durations in central collisions between 1 and  $\sim 3$  fm/c followed by mixed phases lasting 3-4 fm/c.

We are now in position to calculate the desired survival probabilities. The evolution equation for the number of each charmonium species  $i$  ( $i = J/\Psi, \Psi', \chi$ ) in the system at time  $\tau$  reads

$$\frac{dN_i(\tau)}{d\tau} = -\Gamma_{diss}^i N_i(\tau) . \quad (8)$$

The destruction rate  $\Gamma_{diss}^i$  is specified according to the phase and temperature  $T(\tau)$  of the system,

$$\Gamma_{diss}^i = \begin{cases} \Gamma_{QG}^i , & T > T_c \\ f\Gamma_H^i + (1-f)\Gamma_{QG}^i , & T = T_c \\ \Gamma_H^i , & T < T_c \end{cases} \quad (9)$$

(with  $f$  given by eq. (7), and  $\Gamma_{QG}^i$  and  $\Gamma_H^i$  from Sects. III A and III B). Eq. (8) is readily integrated to obtain the survival probability at time  $\tau$  during the collision,

$$\mathcal{S}_{QG+H}^i(\tau) = e^{-\int_0^\tau \Gamma_{diss}^i(\tau') d\tau'} . \quad (10)$$

The final survival probability,  $\mathcal{S}_{QG+H}$ , relevant for experimental observables is simply given by the value of  $\mathcal{S}_{QG+H}^i(\tau)$  at the moment of thermal freezeout,  $\tau_{fo}$ , where all hadrons cease to interact. Note that  $\mathcal{S}_{QG+H}^i$  depends on the impact parameter  $b$  of the collision through the space-time evolution of the system.

Fig. 4 shows  $J/\Psi$  and  $\Psi'$  survival probabilities at full SPS and RHIC energies (central collisions,  $N_{part} = 360$ ) as a function of time with a value of  $g = 1.7$  for the strong coupling constant (which provides reasonable agreement with SPS data to be discussed below). Most of the suppression originates from the plasma and mixed phase, with hadronic

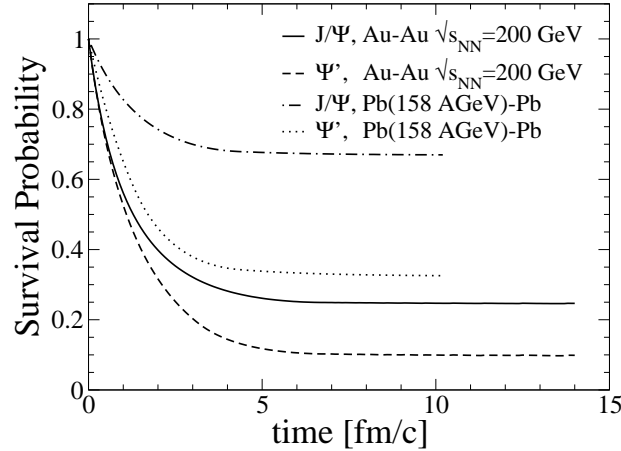


FIG. 4: Survival probability of  $J/\Psi$  (dot-dashed line: SPS, solid line: RHIC) and  $\Psi'$  (dotted line: SPS, dashed line: RHIC) for central collisions ( $N_{part} = 360$ ) as a function of time. The curves are obtained upon integration of the dissociation rates calculated in Sect. III A and III B.

effects playing little role. The destruction of  $\Psi'$  states (dotted line) under SPS conditions is far from complete (with a fraction of about 1/3 remaining at the end of the collision); this may in fact point at a lack of our understanding of the hadronic  $\Psi'$  interactions, since (i) it will cause problems in describing the measured  $\Psi'/\Psi$  ratio (see Sect. V B below), and (ii) it appears to be incompatible with recent lattice calculations [29] which exhibit a dissolution of the  $\Psi'$  in the hadronic phase (although no statement about timescales could be made). As expected, the overall dissociation is much stronger at RHIC (full line:  $J/\Psi$  dashed line:  $\Psi'$ ), although 25% of primordial  $J/\Psi$ 's endure the evolution.

For a combination of the (suppressed) primordial production with the statistical one (to be discussed in the next section) it is necessary to convert the survival probabilities into absolute yields. At a given impact parameter  $b$ , the number of  $J/\Psi$  mesons initially produced in hard parton-parton collisions is

$$N_{J/\Psi}^0(b) = \sigma_{pp}^{J/\Psi} AB T_{AB}(b) . \quad (11)$$



There are no data on charmonium production in  $p$ - $p$  in the RHIC energy regime (to date, the highest  $\sqrt{s}$  energies available lie around  $\sqrt{s_{NN}} \simeq 40$  GeV). Thus we have to rely on extrapolations. The so-called Schuler parameterization [45] – a phenomenological fit based on low-energy systematics – leads to  $\sigma_{pp}^{J/\Psi}(\sqrt{s} = 200 \text{ GeV}) = 1.0 \mu\text{b}$ . On the other hand, the simple ansatz  $\sigma_{pp}^{J/\Psi} = f\sigma_{pp}^{c\bar{c}}$  with  $f \simeq 0.025$  [46] together with PYTHIA extrapolations of open-charm production (see also next section), which also reproduces low-energy data, gives  $\sigma_{pp}^{J/\Psi}(\sqrt{s} = 200 \text{ GeV}) = 8.75 \mu\text{b}$ . This obviously introduces appreciable uncertainties in establishing the  $J/\Psi$  yield arising from primordial production mechanisms.

In addition, higher charmonium states (in particular  $\Psi'$  and various  $\chi$  states) contribute significantly to the measured  $J/\Psi$  abundance via strong and electromagnetic decays (feeddown). Since these resonances have different hadronic and plasma cross sections, the distinction between prompt and secondary  $J/\Psi$ 's has to be made. Consequently, the total number of  $J/\Psi$ 's stemming from primordial charmonium states after nuclear, plasma and hadronic suppression, becomes

$$N_{J/\Psi}^{dir}(b) = \sigma_{pp}^{J/\Psi} ABT_{AB}(b) \mathcal{S}_{nuc} \left[ 0.6 \mathcal{S}_{QG+H}^{J/\Psi} + 0.08 \mathcal{S}_{QG+H}^{\Psi'} + 0.32 \mathcal{S}_{QG+H}^{\chi} \right] . \quad (12)$$

#### IV. STATISTICAL PRODUCTION OF $J/\Psi$

Besides direct  $J/\Psi$  production, another source of  $J/\Psi$ 's has been attributed to thermal (or statistical) production in several variants. We here adopt the approach put forward in Refs. [12, 14], where statistical coalescence of primordial  $c$  and  $\bar{c}$  quarks is assumed to predominantly occur at the hadronization transition. The relative abundances of charmed particles are then determined by hadronic thermal weights at the critical temperature  $T_c$ , whereas the absolute number is specified by the amount of charm quarks created in the early (hard)  $N$ - $N$  collisions.

Starting from the ideal gas expression for the thermal number density of particle species  $i$ ,

$$n_i = \frac{d_i}{2\pi^2} \int_0^\infty p^2 dp \left[ \exp \left( \frac{\sqrt{p^2 + m_i^2} - \mu_i}{T} \right) \pm 1 \right]^{-1} \quad (13)$$

( $d_i$ : spin-isospin degeneracy), the total number densities of open and hidden charm particles follow as

$$n_{op} = \sum n_i , \quad i = D, D^*, \dots \quad (14)$$

$$n_{hid} = \sum n_j , \quad j = \eta_c, \Psi, \dots . \quad (15)$$

The pertinent chemical potentials  $\mu_i$  encode conserved charges (*net* baryon, strangeness and charm numbers), and depend on the collision energy of the system (in accord with the hadro-chemistry). At  $T_c$  we match the densities, Eqs. (14) and (15), to the available number of open charm pairs,  $N_{c\bar{c}}$ , by introducing an effective fugacity factor  $\gamma_c = \gamma_{\bar{c}}$  for both charm and anticharm quarks. Since  $N_{c\bar{c}}$  is typically a small number (*e.g.*,  $\sim 0.2$  for central  $Pb$ - $Pb$  at SPS), exact charm conservation within the canonical ensemble formalism [47, 48] is mandatory, leading to

$$N_{c\bar{c}} = \frac{1}{2} \gamma_c n_{op} V_H \frac{I_1(\gamma_c n_{op} V_H)}{I_0(\gamma_c n_{op} V_H)} + \gamma_c^2 n_{hid} V_H . \quad (16)$$

Here,  $V_H$  denotes the fireball volume from the previous section, cf. Eq. (6), at the moment when hadronization is complete. The actual value of  $N_{c\bar{c}}$  depends on both collision energy and impact parameter. The former dependence is inferred from PYTHIA computations [49] upscaled by an empirical  $K$ -factor of around 5 extracted from a best fit to existing  $pN$  and  $\pi N$  data [50]. As discussed in the previous section, the extrapolation into the RHIC regime bears appreciable uncertainty. *E.g.*, next-to-leading order pQCD calculations [51] for full RHIC energy give  $\sigma_{pp}^{c\bar{c}} \sim 350 \mu\text{b}$ , to be compared to  $\sigma_{pp}^{c\bar{c}} \sim 570 \mu\text{b}$  from the PYTHIA estimate. Both extrapolations are in line with recent indirect measurements from PHENIX [52] in  $Au$ - $Au$  at  $\sqrt{s_{NN}} = 130$  GeV, where single-electron  $p_t$ -spectra have been used to infer  $\sigma_{pp}^{c\bar{c}} = 380 \pm 60(\text{stat}) \pm 200(\text{syst}) \mu\text{b}$ , to be compared to  $\sigma_{pp}^{c\bar{c}} \sim 320 \mu\text{b}$  within the PYTHIA extrapolation.

By construction, statistical charmonium production is only active for  $c$  and  $\bar{c}$  that emerge from a deconfined environment prior to recombination. Therefore, in peripheral collisions where the initial volume is only partially filled with the QGP phase, only a fraction of the  $c\bar{c}$  pairs is available for coalescence. Accordingly, the hadronization volume which enters Eq. (16) arises from the initial volume in the plasma phase after expansion.

The thermal equilibrium (but chemical off-equilibrium)  $J/\Psi$  yield takes the form  $\langle J/\Psi \rangle = \gamma_c^2 V_H n_{J/\Psi}$ . The former expression is valid if (anti-) charm quarks are kinetically equilibrated, *i.e.*, the momentum distribution of  $c$  and  $\bar{c}$  quarks is thermal which is questionable under SPS and even RHIC conditions. We therefore implement the following correction: we introduce a thermalization time  $\tau_{eq}$  for  $c$  and  $\bar{c}$  quarks, approximated as  $\tau_{eq} = 1/n\sigma$ , where  $n$  is the total density of quark- and gluon-quasiparticles in the system, and  $\sigma$  is the elastic scattering cross section for the processes  $g(q, \bar{q}) + c(\bar{c}) \rightarrow g(q, \bar{q}) + c(\bar{c})$ . Within a relaxation time approach, the relative reduction  $\mathcal{R}$  in thermal  $J/\Psi$  formation is then estimated as

$$\mathcal{R} = \left[ 1 - \exp \left( - \int_0^{\tau_H} \frac{d\tau}{\tau_{eq}} \right) \right], \quad (17)$$

where  $\tau_H$  is the time at which hadronization is completed. Our expression for the number of  $J/\Psi$ 's produced at the hadronization transition by coalescence of a  $c$  and  $\bar{c}$  quark is thus modified according to

$$\langle J/\Psi \rangle = \gamma_c^2 V_H n_{J/\Psi} \mathcal{R}. \quad (18)$$

We have checked that our results do not change significantly with the definition adopted for  $\tau_H$ . The latter is not a sharply defined quantity since the mixed phase lasts for a few fm/c. However, smaller values of  $\tau_H$  (*e.g.*, if taken in the middle of the mixed phase) lead to smaller volumes and larger  $\gamma_c$  implying an increase in thermal  $J/\Psi$  production which, in turn, is (partially) compensated by a less degree of thermalization through a smaller value of  $\mathcal{R}$ .

The charmonia statistically produced at the hadronization transition are still subject to reinteractions in the hadronic phase, so that their final contribution to the observed  $J/\Psi$  yield is accounted for via

$$N_{J/\Psi}^{th} = \gamma_c^2 V_H \left[ n_{J/\Psi} \mathcal{S}_H^{J/\Psi} + \sum_X BR(X \rightarrow J/\Psi) n_X \mathcal{S}_H^X \right] \mathcal{R}, \quad (19)$$

where the summation is carried over the charmonium states  $X$  with finite decay branching into  $J/\Psi$ 's (feeddown).

## V. TWO-COMPONENT MODEL AT THE SPS

By combining the two sources of charmonium as discussed in the previous sections, we now turn to applications to heavy-ion collisions at SPS energy ( $\sqrt{s_{NN}} = 17.2$  GeV). We first address the observable that has drawn the most attention, *i.e.*, the centrality dependence of the  $J/\Psi$  yield, but also investigate the  $\Psi'/\Psi$  ratio.

### A. Centrality dependence of $J/\Psi$ production

One of the important findings of the NA38 and NA50 experiments at CERN is that the  $J/\Psi$  yield in  $p$ - $p$  and  $p$ - $A$  collisions with light and heavy targets is well explained by hard production coupled with nuclear absorption, cf. Sect. II. Thus, any attempt to describe the  $J/\Psi$  yield in heavy-ion collisions must reproduce this feature. In particular, for very peripheral collisions involving only a few participant nucleons, the  $J/\Psi$  suppression pattern should coincide with nuclear absorption.

In our two-component model, the total number of observed  $J/\Psi$ 's is the sum of direct and statistical production according to Eqs. (12) and (19), respectively,

$$N_{J/\Psi} = N_{J/\Psi}^{dir} + N_{J/\Psi}^{th}. \quad (20)$$

Both terms on the right-hand side depend on centrality (via the impact parameter  $b$ ) and collision-energy. For very peripheral collisions in the SPS regime, the initial conditions are not energetic enough to induce a transition into the QGP. Therefore, the  $J/\Psi$  source of statistical recombination at  $T_c$  is absent; at the same time the hadronic interactions are not frequent enough to induce sizable effects. Thus, for large impact parameters our approach is consistent with the NA38/NA50 results.

For a detailed comparison with data, we need to evaluate the ratio  $B_{\mu\mu}\sigma^{J/\Psi}/\sigma^{DY}$  commonly displayed by NA38/NA50 as a function of transverse energy deposited in their calorimeter. Following the treatment outlined in Sect. II, we convolute our theoretical results (which are obtained as a function of impact parameter  $b$ ) with the probability distribution  $\mathcal{P}(E_T, b)$ . This amounts to replacing the numerator in Eq. (3) by the expression

$$B_{\mu\mu}\sigma_{J/\Psi}(E_T) = B_{\mu\mu}\sigma_{J/\Psi}^{pp} \int d^2b \mathcal{P}_{AB}(E_T, b) \left[ \mathcal{S}_{nuc} \mathcal{S}_{QG+H} + N_{J/\Psi}^{th}/(\sigma_{J/\Psi}^{pp} AB T_{AB}(b)) \right] T_{AB}(b). \quad (21)$$

Our results are compared to NA38/NA50 data in Fig. 5 for both the  $S(200 \text{ AGeV})$ - $U$  (left panel) and  $Pb(158 \text{ AGeV})$ - $Pb$  (right panel) system. At all centralities, direct production (dashed line) prevails over the thermal component

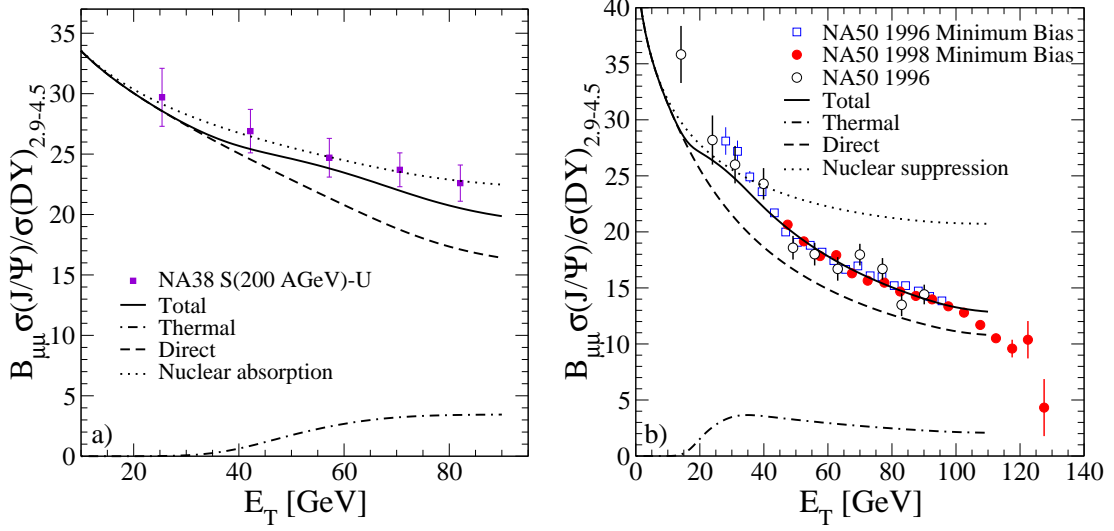


FIG. 5: Results on  $J/\Psi$  production within the two-component model as a function of centrality compared to NA38 [5] (left panel) and NA50 data [24] for  $S(200 \text{ AGeV})$ - $U$  and  $Pb(158 \text{ AGeV})$ - $Pb$ , respectively. Dotted lines: direct production with nuclear absorption alone; dashed lines: direct production subject to nuclear, QGP and hadronic absorption; dot-dashed lines: statistical (thermal) production from a hadronizing QGP including hadronic dissociation; solid lines: combined direct and statistical yield (sum of dashed and dot-dashed lines).

(dot-dashed line). The latter sets in once a QGP starts forming, which, in turn, requires a stronger QGP suppression of the direct component than without the thermal contribution. The adjustment of the only free parameter (strong coupling constant  $g = 1.7$ ) to the most central  $Pb$ - $Pb$  data allows for a satisfactory reproduction of the centrality dependence for this system. For  $S$ - $U$  collisions, the results are somewhat on the low side. Note that in our approach the “drop” in the  $Pb$ - $Pb$  data around  $E_T \simeq 40 \text{ GeV}$  is a combination of a rather strong QGP suppression coupled with the onset of thermal production.

In its present form, our model does not capture the appearance of the “second drop” in the data for the most central  $Pb$ - $Pb$  collisions at  $E_T > 100 \text{ GeV}$ . In fact, the maximum transverse energy in our description is at  $E_T^{max} = E_T(b=0) = 100 \text{ GeV}$ , well below the experimental limit which extends up to  $E_T \simeq 125 \text{ GeV}$ . It has been suggested that these features are associated with transverse energy fluctuations [53, 54, 55] and/or trigger energy losses [56], and are thus not necessarily related to a shortcoming in an underlying (microscopic) model description of  $J/\Psi$  production (suppression).

Let us first address the  $E_T$  fluctuations. From the minimum bias ( $MB$ ) event distribution of transverse energy,  $dN/dE_T$ , as measured in the NA50 apparatus [57] one finds a rapid falloff beyond  $E_T = 100 \text{ GeV}$ , the so-called “knee” of the distribution. The tail of the latter, which reaches beyond  $E_T = 100 \text{ GeV}$ , is associated with fluctuations in transverse energy at fixed geometry for  $b = 0$ . Events which fluctuate beyond  $E_T > 100 \text{ GeV}$  contain a larger initial entropy and consequently a hotter initial temperature and a longer plasma phase which implies additional  $J/\Psi$  suppression (charm and  $J/\Psi$  production being a hard process are not coupled to fluctuations in the “soft” sector). To account for this phenomenon [53], we replace in our calculations the total entropy at fixed impact parameter,  $S_{tot}(b)$ , by

$$S_{tot}(b) \rightarrow S_{tot}(b) \frac{E_T}{E_T(b)}. \quad (22)$$

This does not affect our results for  $E_T \leq E_T(b=0) \simeq 100 \text{ GeV}$ , but beyond the knee of the distribution,  $S_{tot}$  is enhanced by the factor  $E_T/E_T(b)$ . The effect of this modification is shown in the left panel of Fig. 6, where we compare our calculations to data on  $J/\Psi$  production normalized to the minimum bias  $E_T$ -distribution<sup>1</sup>. Obviously,

<sup>1</sup> This way of normalizing the data has the advantage of the much better statistics for  $dN/dE_T$  as compared to the Drell-Yan sample. On

the description of the observed turnover for  $E_T > 100$  GeV is improved by inclusion of  $E_T$ -fluctuations (cf. dashed versus dot-dashed curve), but does not suffice to quantitatively explain the data. Note that this effect relies on additional  $J/\Psi$  suppression in both the direct component (due to stronger suppression) and the thermal component (due to a larger hadronization volume weakening the enhancement induced by the canonical-ensemble treatment).

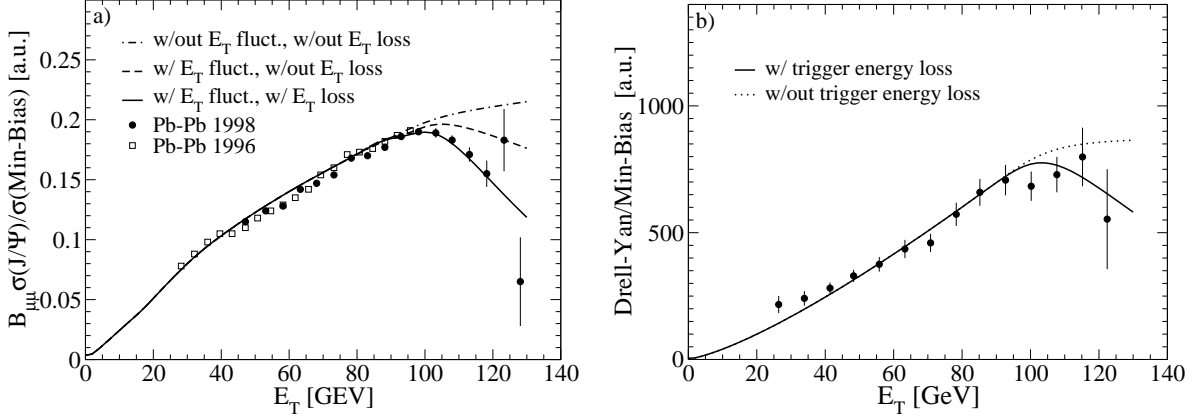


FIG. 6: Left panel:  $J/\Psi$  over Minimum Bias ( $MB$ ) ratio compared to calculations within the two-component model (dot-dashed curve) with additional inclusion of effects due to  $E_T$  fluctuations (dashed curve) and trigger energy loss (solid curve). Right panel: impact of a model for trigger energy loss on the Drell-Yan ( $DY$ ) over  $MB$  ratio. All data points are from NA50 for  $Pb(158 \text{ AGeV})\text{-}Pb$  [57].

However, as has been suggested by Capella *et al.* [56], there might be an additional feature in the large  $E_T$ -region unrelated to  $J/\Psi$  physics, which can be gleaned from the Drell-Yan ( $DY$ ) over minimum bias data, cf. right panel of Fig. 6. The theoretical ratio  $DY/MB$  (dotted line) computed by NA50 flattens out at large values of  $E_T > 100$  GeV, whereas the data seem to indicate a slight turnover [57]. The argument [56] to explain this turnover (which equally applies to the  $J/\Psi$  event sample) is that for  $DY$ -events the hadronic transverse energy deposited in the calorimeter is slightly reduced as compared to the corresponding  $MB$ -events due to triggering on the  $DY$  (or  $J/\Psi$ ) pair ( $2.9 \text{ GeV} < M_{DY} < 4.5 \text{ GeV}$ ). As elaborated in Ref. [56], a rough estimate of this effect is obtained by rescaling the amount of transverse energy in the  $J/\Psi$  and  $DY$  events according to

$$E_T(b) \rightarrow E_T(b) \frac{N_{part} - 2}{N_{part}}. \quad (23)$$

When incorporated into the  $E_T$ - $b$  correlation, Eq. (2), this rather small loss in  $E_T$  entails a surprisingly large drop of about  $\sim 20\text{-}30\%$  in the tail of the  $(DY/MB)$  distribution, cf. Fig. 6 (solid curve in the right panel).

Returning to the  $(J/\Psi)/MB$  ratio (left panel of Fig. 6), we see that the combined effect of  $E_T$  fluctuations and trigger energy loss, implemented within our microscopic model, gives a satisfactory description of the experimental findings. This also holds true for the more common representation of the data via the  $J/\Psi/DY$  ratio, cf. Fig. 7. Note that in this case the trigger energy-loss correction only applies to the “ $MB$ ” data sets, which have been extracted using a theoretical expression for the  $MB/DY$  ratio according to

$$\left(\frac{J/\Psi}{DY}\right)_{MB \text{ analysis}} = \left(\frac{J/\Psi}{MB}\right)_{exp} \left(\frac{MB}{DY}\right)_{th}. \quad (24)$$

## B. $\Psi'/\Psi$ ratio

In  $p$ - $p$  collisions in the SPS energy regime, the ratio of produced  $\Psi'$  to  $J/\Psi$  mesons amounts to a value of about 15%, which persists for  $p$ - $A$  collisions as nuclear absorption affects both charmonium states in practically the same

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the other hand, since  $E_T$  production is governed by soft physics (which essentially scales with the number of participants rather than the number of  $N$ - $N$  collisions), the characteristic features of  $J/\Psi$  suppression are not readily discernible.

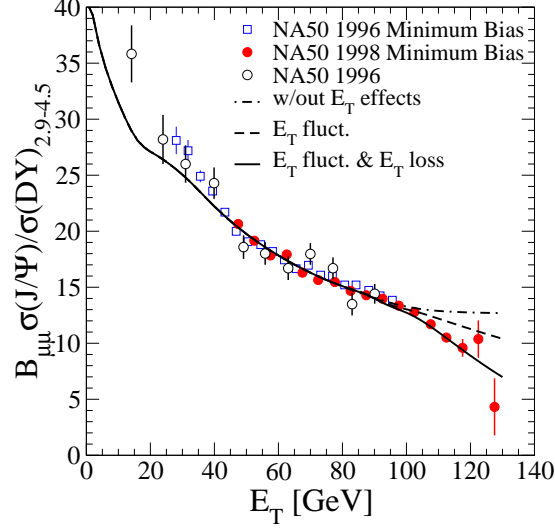


FIG. 7: Results of the two-component model without (dot-dashed line) and with additional inclusion of transverse energy fluctuations (dashed line) and trigger energy loss (full line), for the centrality dependence of the  $B_{\mu\mu} \sigma^{J/\Psi} / \sigma^{DY}$  ratio in  $Pb(158 \text{ AGeV})$ - $Pb$  collisions.

way, cf. Sect. II. A marked deviation from this behavior has been observed in  $S(200 \text{ AGeV})$ - $U$  and  $Pb(158 \text{ AGeV})$ - $Pb$  collisions, with an onset at rather low centralities. Remarkably, for central collisions, the  $\Psi'/\Psi$  ratio does not go to zero, but rather levels off at a value of around 3-4%. This is contrary to the naive expectation that  $\Psi'$  mesons, due to their much smaller binding energy than the  $J/\Psi$  states, are significantly more suppressed. In Ref. [58] it has been suggested that, under the premise that most  $\Psi'$  states are dissolved in the QGP, their abundance is regenerated in the hadronic phase from remaining  $\Psi$  states as a consequence of chiral symmetry restoration, via the process  $\Psi + \pi \rightarrow \Psi'$ . The interaction was assumed to be mediated through  $\sigma(500)$  meson exchange, the mass of the latter approaching the pion mass thereby substantially enhancing  $\Psi'$  formation. From another point of view, the fact that the value of 4% reflects the thermal ratio of  $\Psi'/\Psi$  at a temperature of  $T = 170 \text{ MeV}$  (with little latitude), has been put forward in ref. [12] as evidence for statistical charmonium production at the hadronization transition.

Within our two-component model as laid out above, the  $\Psi'/\Psi$  ratio follows without further assumptions. The results for both  $S$ - $U$  and  $Pb$ - $Pb$  systems are compared to NA38/50 data in Fig. 8. The discrepancy with experiment

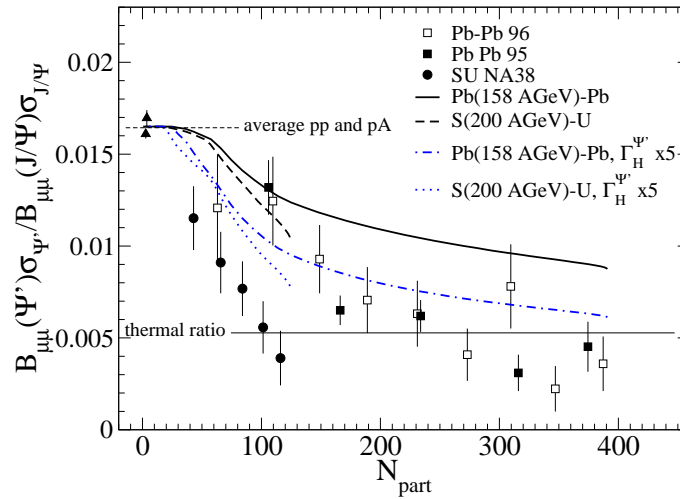


FIG. 8: Our calculations for the centrality dependence of  $\Psi'/\Psi$  compared to data from NA38/50. The solid and dashed line are for  $S(200 \text{ AGeV})$ - $U$  and  $Pb(158 \text{ AGeV})$ - $Pb$ , respectively, using hadronic  $\Psi'$  dissociation cross sections obtained by geometric scaling of the  $J/\Psi$  one, cf. Fig. 3. The dashed and dotted lines are the corresponding results when artificially increasing the  $\Psi'$  cross sections by another factor of 5.

is rather significant, especially for  $S(200 \text{ AGeV})$ - $U$ . Since in the latter reaction QGP effects are not expected to play a pronounced role, it seems that the deficiency in our description has to be assigned to the hadronic phase, *i.e.*, an underestimation of the hadronic cross sections for the  $\Psi'$ . Indeed, an artificial increase of this quantity by, say, a factor of 5 clearly improves the agreement with the data. We have checked that such an increase in the  $\Psi'$  hadronic cross sections has negligible impact on the  $\Psi/DY$  ratio as plotted in Fig. 7 (the  $\Psi'$  contributes maximally 8% to the observed  $J/\Psi$  yield).

As mentioned before, recent lattice calculations indicate a dissolution of the  $\Psi'$  in a static environment well below the phase transition temperature, due to the lowering of the  $D\bar{D}$  continuum threshold below the in-medium  $\Psi'$  mass. In a hadronic model framework, this can be implemented by an in-medium reduction of the  $D$ -meson masses, which has been motivated in Ref. [59] by chiral restoration arguments inducing a lowering of the light-quark mass within the  $c\bar{q}$  and  $\bar{c}q$  states. We have investigated this possibility within the hadronic approach employed here, and found a strong sensitivity to the detailed modeling of the light-quark related portion of the  $D$ -meson masses. In fact, if the lifetimes of the charmonium states become comparable to duration of the fireball expansion, one needs to account for the reverse reaction of charmonium formation (as required by detailed balance), which is beyond the scope of this paper<sup>2</sup>.

A more controlled way to assess hadronic medium effects in charmonium dissociation should be provided by constituent quark (exchange-) models incorporating both phenomenological confinement potentials as well as properties of chiral symmetry breaking [33, 38, 60, 61]. This will be addressed elsewhere [62].

## VI. EXCITATION FUNCTION AND PREDICTIONS FOR RHIC

An essential part of the experimental program at RHIC is again on (penetrating) electromagnetic probes. The PHENIX detector will provide accurate dilepton data via both the (forward) muon arms as well as electron identification in the central region. The results on charmonium should allow for stringent constraints on models. At full RHIC energy, standard extrapolations predict an open-charm production that is about two orders of magnitude larger than in the SPS regime, entailing a substantial increase in the statistical recombination mechanism for charmonia. At the same time, direct (hard) charmonium production, albeit also enhanced by presumably a similar factor as open charm, ought to be more strongly suppressed due to longer and initially hotter QGP phases.

A quantitative comparison between SPS and RHIC within our two-component model is performed in Fig. 9 where the final (observed) number of  $J/\Psi$ 's, normalized to the number  $J/\Psi$ 's remaining after nuclear absorption,  $N_{J/\Psi}^{nuc}$ , is displayed for central collisions as a function of the fireball evolution time.

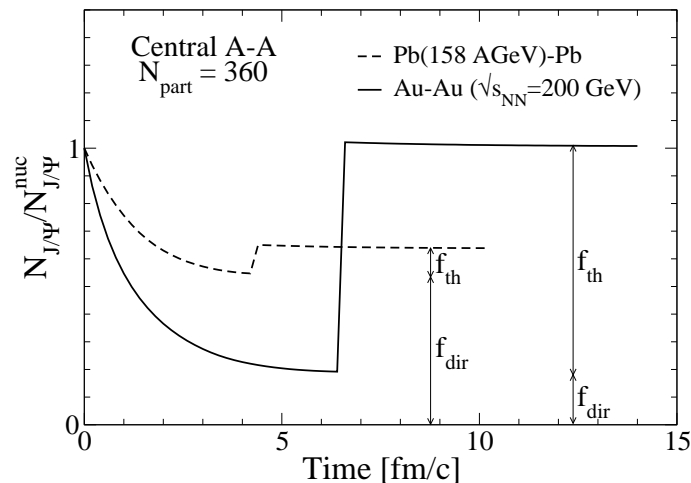


FIG. 9: Time dependence of the ratio  $N_{J/\Psi}/N_{J/\Psi}^{nuc}$  at SPS (dashed line) and RHIC (full line) for central collisions with  $N_{part} = 360$ , where  $N_{J/\Psi}^{nuc}$  is the number of  $J/\Psi$ 's remaining after nuclear absorption. The respective fractions of direct ( $f_{dir}$ ) and thermal ( $f_{th}$ ) yields are indicated by the arrows.

<sup>2</sup> Note that with an increase by a factor of 5 for  $\Psi'$  dissociation rate over the results shown in Fig. 3 (as applied in Fig. 8) the  $\Psi'$  lifetimes in the vicinity of  $T_c$  are indeed close to the expansion time of the hadronic phase.

The freezeout value for this ratio increases by about 50% going from SPS to RHIC (from 0.65 to about 1). More dramatically, the composition in terms of underlying sources is very different: whereas at SPS the (suppressed) direct yield dominates,  $J/\Psi$ -mesons at RHIC originate to  $\sim 80\%$  from thermal production (being proportional to  $(N_{c\bar{c}})^2$ ). The upward jump of the two curves in Fig. 9 is located at the respective end of the mixed phase,  $\tau_H$ , where in our approximation all thermal production is concentrated. As elucidated in Sect. IV, the final results are not sensitive to the exact production time within the mixed phase.

Since the phenomenological extrapolations for absolute numbers of primordial  $J/\Psi$  and charm-quark production up to RHIC energies, which are input parameters to our calculations, are beset with appreciable uncertainties, it is desirable to define a quantity which reduces this sensitivity. Therefore, we show in Fig. 10 the predictions of our two-component approach for centrality dependencies of the ratio  $N_{J/\Psi}/N_{c\bar{c}}$ , which, in anticipation of open-charm measurements at RHIC, also has the virtue of being composed of experimental observables<sup>3</sup>. Whereas at SPS energies

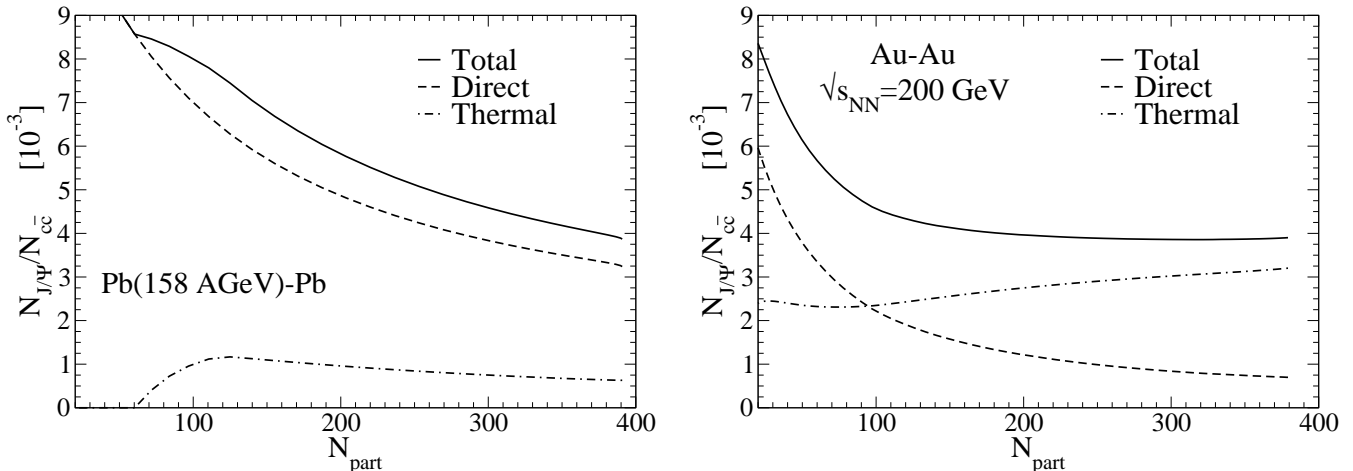


FIG. 10: Comparison of the centrality dependence of the  $N_{J/\Psi}/N_{c\bar{c}}$  ratio at SPS and RHIC full energy (full line). The direct (thermal) contributions are shown separately, respectively in the dashed (dot-dashed) lines.

this quantity exhibits a monotonous decrease with increasing number of participants (left panel of Fig. 10), it saturates already for rather peripheral collisions at full RHIC energy. Note that the decrease of the thermal component for  $N_{part} \geq 100$  at SPS is caused by canonical ensemble effects, while at RHIC statistical production, for the most part, proceeds in the grand-canonical limit entailing a smooth increase with centrality. Our approach thus clearly discriminates between standard  $J/\Psi$  suppression as opposed to thermal regeneration at full RHIC energy.

It is therefore important (and experimentally feasible at RHIC) to map out the transition between the regimes of predominantly direct to thermal charmonium production, as has been first pointed out in Ref. [19]. In Fig. 11 we present an updated prediction of the excitation function for  $N_{J/\Psi}/N_{c\bar{c}}$  ratio<sup>4</sup>. The ratio exhibits a nontrivial minimum structure around  $\sqrt{s_{NN}} \simeq 40$  GeV, which is a marked feature of the interplay between hard and thermal production (assuming no anomalies in open-charm production, cf. Ref. [64]).

For practical purposes, it is also of interest to convert our results into an absolute number of  $J/\Psi$ 's predicted for RHIC. Keeping in mind the uncertainties mentioned above, we find that the combined statistical and direct  $J/\Psi$  yield amounts to rapidity density  $dN/dy \sim 1 \times 10^{-2}$  for central collisions at full RHIC energy.

## VII. COMPARISON TO OTHER WORKS

The evaluation of  $J/\Psi$  dissolution in a QGP dates back to the late 70's / early 80's and has been discussed in many facets. In our approach, we have combined an in-medium reduced binding energy with parton-induced destruction,

<sup>3</sup> However, this ratio may still be sensitive to, *e.g.*, shadowing effects: since the  $J/\Psi$  yield, being mostly thermal, goes as  $N_{c\bar{c}}^2$  a modification of  $c\bar{c}$  production due to nuclear shadowing does not cancel out in this ratio.

<sup>4</sup> As compared to our earlier results, we here also incorporate the corrections from feeddown of excited charmonia and from hadronic suppression. Within the uncertainty band quoted in Ref. [19], the results agree.

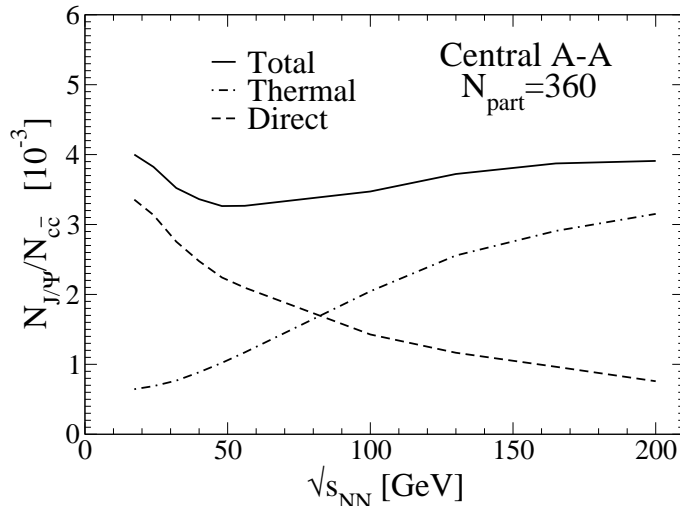


FIG. 11: Excitation function of the  $N_{J/\Psi}/N_{c\bar{c}}$  ratio (full line). The interplay between the direct contribution (dashed line) and the thermal component (dot-dashed line) results in a minimum in the excitation function around  $\sqrt{s_{NN}} \simeq 40$  GeV.

which we have treated in quasifree approximation due to small (if any) binding energies of the various charmonium states. The latter feature is also the main origin of the rather strong suppression we find using rather moderate values of the strong coupling constant,  $\alpha_s \simeq 0.3$ , which, of course, is to be regarded as an effective parameter which we adjusted to describe the NA50 data. Due to a  $\sim 20\%$  contribution from statistical  $J/\Psi$  production at SPS energies, our QGP suppression required to reproduce the NA50 data is stronger than in calculations based on the suppression effect alone. On the other hand, within the quasifree approximation, the discrimination between different charmonia is less pronounced than within “threshold”-type approaches where higher charmonium states are completely suppressed once the QGP temperature exceeds the relevant value.

Concerning the evaluation of thermal (statistical) recombination, our approach parallels the one initiated in Refs. [12, 14], which is based on an open-charm abundance generated by hard production, available for recombination at  $T_c$ . The main difference to these analyses is that we do not invoke an enhancement of open-charm production beyond standard extrapolations of  $N-N$  collisions (also, we include corrections from incomplete thermalization of charm quarks in relaxation time approximation). Such an enhancement is needed if one aims at describing the NA50 data in terms of thermal production alone (assuming that directly produced charmonia have been completely dissolved). The NA50 suppression pattern towards more peripheral collisions then emerges as a canonical ensemble effect, requiring “anomalous” open-charm enhancement factors of around  $\sim 5$  [16]. Theoretically, such an increase in hard production in a heavy-ion environment is not easily justified. Also, the NA38 findings on the intermediate-mass dimuon excess [65] allow for a maximal open-charm enhancement of a factor of  $\sim 3$  for central collisions (less for peripheral)<sup>5</sup>. Furthermore, imposing statistical production on small-size systems (or even  $p-A$  reactions) eventually contradicts nuclear absorption systematics.

Finally we refer to Ref. [18], where the  $J/\Psi$  abundance in an expanding QGP has been evaluated using rate equations with gluon photodissociation and its reverse reaction as microscopic input. The space-time evolution in ref. [18] occurs entirely within the QGP phase, and in-medium modifications to the  $J/\Psi$  binding energy have not been applied. Consequently, the time evolution of  $J/\Psi$  formation in that approach is quite different from our results, leaning towards early production [68], which also entails substantially larger results for the final  $N_{J/\Psi}/N_{c\bar{c}}$  ratio, reaching values of up to  $20\text{--}30 \times 10^{-3}$  for central  $Au-Au$  at  $\sqrt{s_{NN}}=200$  GeV. This is a factor of 5-8 larger than our estimates, cf. Fig. 11.

<sup>5</sup> In our picture, this excess is attributed to thermal radiation [66, 67].



## VIII. CONCLUSIONS AND OUTLOOK

In summary, we have developed a two-component model for charmonium production in heavy-ion collisions within a comprehensive thermal evolution scenario. The two sources of charmonium production are (i) “direct”  $J/\Psi$ ’s, arising from primordial  $N$ - $N$  collisions, subjected to nuclear, QGP and hadronic suppression, as well as (ii) statistical recombination of  $c$  and  $\bar{c}$  quarks at the hadronization transition subjected to hadronic dissociation only. The QGP dissociation of contribution (i) has been evaluated using in-medium charmonium binding energies which led us to introduce “quasifree” destruction as the dominant suppression mechanism. Inelastic hadronic interactions have been estimated within earlier proposed effective lagrangian approaches, and turned out to give very moderate corrections (up to 10% for excited charmonium states). Contribution (ii) has been based on open-charm abundances as inferred from  $N$ - $N$  collisions (without “anomalous” enhancement), and incomplete thermalization has been incorporated via a relaxation time approximation. Taken together, with an effective strong coupling constant as a single parameter, the measured centrality dependence of  $J/\Psi$  production at the SPS can be reasonably well described. A potential discrepancy has been identified in the  $\Psi'/\Psi$  ratio, which we believe to reside in shortcomings of the calculations for hadronic  $\Psi'$  dissociation. The latter is difficult to assess in purely hadronic models and might well be underestimated.

We have extrapolated our approach to higher energies. The pertinent excitation function for the  $N_{J/\Psi}/N_{c\bar{c}}$  ratio in central collisions exhibits a non-trivial minimum structure around  $\sqrt{s_{NN}} \sim 40$  GeV signalling the transition from (predominantly) direct to thermal production [19]. For the centrality dependence of this ratio at full RHIC energy, we predict a rather flat behavior for participant numbers beyond  $N_{part} \simeq 150$ . The absolute number of produced  $J/\Psi$  mesons in central collisions at RHIC turns out to be close to what one expects from nuclear absorption alone, *i.e.*, an 80% QGP suppression is essentially regenerated by thermal production at hadronization.

We emphasize again that the present analysis should also be considered as another step towards establishing a coherent picture of high-energy heavy-ion collisions as proceeding through a thermal evolution including QGP formation. Charmonium production has been linked with other observables such as hadro-chemistry [8, 69], dilepton production at low and intermediate mass [41, 66, 70], etc.

Concerning directions for future work, it will be necessary to reiterate the question of in-medium effects in hadronic dissociation cross sections especially for the lightly bound charmonium states. Here, quark-exchange models appear to be better suited than hadronic ones, allowing for modifications of effective confining potentials and constituent quark masses [38] – possibly related to effects of chiral restoration – on a microscopic level. Furthermore, the NA50 data on  $J/\Psi$  transverse momentum distributions [71] need to be addressed as another test of our approach. It is also mandatory to improve on our schematic treatment of the charm-quark thermalization. Here, valuable information can be expected from experiment in terms of both single- and di-lepton spectra from semileptonic open-charm decays [72, 73].

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